JOURNAL OF GUIDANCE, CONTROL, AND DYNAMICS
Vol. 24, No. 3, May–June 2001

Adaptive Output Feedback Control of an Aeroelastic System with Unstructured Uncertainties

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This paper presents an adaptive control system for the trajectory control and stabilization of an aeroelastic system with unstructured model uncertainties using output feedback. The equations of motion of the chosen aeroelastic system describe the plunge and pitch motion of a structurally nonlinear wing section. A single trailing-edge flap is used for the purpose of control. For feedback only the pitch angle or the plunge displacement is measured. Unlike the linearly parameterized adaptive systems, in this study the structure of model uncertainties is unknown. Adaptive control laws for the trajectory control of the pitch angle and the plunge displacement are derived. The controller has the structure of an inverse (a feedback linearizing) control system. But the unknown function in the inverse control law arising from the uncertainties in the model is compensated using its estimate constructed by a high-gain observer. Simulation results are presented, which show that in the closed-loop system trajectory control of the selected output (pitch angle or plunge displacement) is accomplished and the state vector converges to zero. Furthermore, it is observed that stability and trajectory tracking are preserved in the presence of measurement noise.

Nomenclature

A_x, A_f, A_v, A_0, F_x = system matrices
a = nondimensionalized distance from the midchord to the elastic axis
b_s = semichord of the wing
c_h = structural damping coefficient in plunge caused by viscous damping
c_{ik}, k_i, M_i, b, g, b_0 = model parameters and matrices
c_α = structural damping coefficient in pitch caused by viscous damping
F_v, $k_{n\alpha}$ = nonlinear functions
h, h_r = plunge displacement, reference trajectory
I_α = mass moment of inertia of the wing about the elastic axis
k_h = structural spring constant in plunge
k_α = structural spring constant in pitch
m = mass
U = freestream velocity
x_s, x, q, v = state vectors
x_α = nondimensionalized distance measured from the elastic axis to the center of mass
Y_r = y_r and its derivatives
y, y_r, $\tilde{y}$ = output, reference output, tracking error
α, α_r = pitch angle, reference pitch angle
β = flap deflection
Δa, Δd = uncertain functions
ϵ, λ_c, p_i, d_i = design parameters
η = uncertain function
$\hat{\eta}$, z_i = estimator states
ξ, e = estimation error
ρ = density of air

I. Introduction

AEROELASTIC systems may exhibit a variety of responses including limit-cycle oscillation, flutter, internal resonance,

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Received 8 February 2000; revision received 28 September 2000; accepted for publication 30 September 2000.

and even chaotic motion. For certain wing store configurations researchers have found that fighter aircraft undergo undesirable limit-cycle oscillations. Therefore, active control of aeroelastic systems has been a problem of considerable interest. Several researchers have considered control law synthesis for the flutter suppression.[1−6] Model order reduction and controller design have been considered in Refs. 2 and 3. Design using pole placement technique and root-locus plots for flutter suppression has been presented in Refs. 4 and 5. References 6 and 7 provide flutter suppression using piezoelectric actuators. Digital adaptive control of an aeroelastic model has been considered in Ref. 8. A benchmark active control technique (BACT) wind-tunnel model has been designed, and control algorithms for flutter suppression have been developed at the NASA Langley Research Center.[9] Dynamics and control of nonlinear aeroelastic models have been considered.[10−18] Feedback linearizing control[13,14] and adaptive control[16−18] of an aeroelastic nonlinear structure have been presented. Although the design in Refs. 13 and 14 requires state feedback, adaptive output feedback control has been considered in Refs. 16 and 17. The control law of Ref. 16 is designed using the variable structure model reference adaptive control theory, which requires high-gain feedback. Nonlinear adaptive control of Ref. 17 is based on a backstepping design technique. Although Ref. 17 does not use high-gain feedback, for the synthesis of the controller, filters of large dimensions are needed. Adaptive control law of Ref. 18 requires state feedback. Based on the Euler–Lagrange theory, a stabilizing control law for the BACT model using generalized velocity feedback has been presented.[19]

The adaptive controllers of Refs. 17 and 18 assume that the aeroelastic models are linearly parameterized. However, often the structure of nonlinearities in the model is unknown. In general, for aeroelastic systems it is not easy to model structural and aerodynamic nonlinearities in a parameterized form. For such systems, adaptive control techniques of Refs. 17 and 18 based on linearly parameterized models cannot be applied. In a recent paper, an approach to control of an exact feedback linearizable system[20] with model uncertainties has been presented using a high-gain observer.[21−23] It is interesting to explore the applicability of this approach to aeroelastic systems with modeling error. However, the aeroelastic model considered here is only partially linearizable, and, unlike Ref. 20, it has zero dynamics of nonzero dimension.

The contribution of this paper lies in the design of an adaptive output feedback control law for the control of a nonlinear aeroelastic system. It is assumed that the system includes unstructured uncertainties, which cannot be parameterized. A single trailing-edge control surface is used for the purpose of control. Control laws for the trajectory control of the pitch angle and plunge displacement are derived. For the synthesis of the controller, it is assumed that only the pitch angle or the plunge displacement is measured. The control system is similar to an inverse controller, which includes a modeling error term. An estimate of the modeling error and state variables is constructed using a high-gain observer for synthesis. In the closed-loop system, asymptotic feedback linearization is accomplished, and the pitch angle and the plunge motion asymptotically converge to the origin. Simulation results are obtained, which show asymptotic tracking and stabilization in spite of the presence of model uncertainties. Furthermore, the control system is robust to measurement noise. Unlike the system of Ref. 20, the aeroelastic model is only input-output (partially) linearizable. As such certain arguments in the derivation of the control law differ.

II. Aeroelastic Model and Control Problem

The prototypical aeroelastic wing section is shown in Fig. 1. The governing equations of motion are provided in Ref. 13, which are given by

$$\begin{bmatrix} m & mx_\alpha b_s \\ mx_\alpha b_s & I_\alpha \end{bmatrix}\begin{bmatrix} \ddot{h} \\ \ddot{\alpha} \end{bmatrix} + \begin{bmatrix} c_h & 0 \\ 0 & c_\alpha \end{bmatrix}\begin{bmatrix} \dot{h} \\ \dot{\alpha} \end{bmatrix} + \begin{bmatrix} k_h & 0 \\ 0 & k_\alpha(\alpha) \end{bmatrix}\begin{bmatrix} h \\ \alpha \end{bmatrix} = \begin{bmatrix} -L \\ M \end{bmatrix} \tag{1}$$

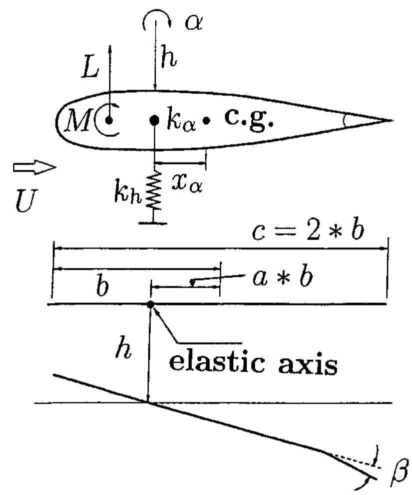


Fig. 1 Aeroelastic model.

In Eq. (1) M and L are the aerodynamic moment and lift. It is assumed that the quasi-steady aerodynamic force and moment are of the form

$$L = \rho U^2 b_s c_{l_\alpha}\left[\alpha + (\dot{h}/U) + \left(\tfrac{1}{2} - a\right)b_s(\dot{\alpha}/U)\right] + \rho U^2 b_s c_{l_\beta}\beta$$
$$M = \rho U^2 b_s^2 c_{m_\alpha}\left[\alpha + (\dot{h}/U) + \left(\tfrac{1}{2} - a\right)b_s(\dot{\alpha}/U)\right] + \rho U^2 b_s^2 c_{m_\beta}\beta \tag{2}$$

where c_{l_α} and c_{m_α} are the lift and moment coefficients per angle of attack and c_{l_β} and c_{m_β} are lift and moment coefficients per control surface deflection. Although, other forms of nonlinear spring stiffness associated with the pitch motion can be considered, for purposes of illustration, the function $k_\alpha(\alpha)$ is considered as a polynomial nonlinearity given by

$$k_\alpha(\alpha) = 2.82(1 - 22.1\alpha + 1315.5\alpha^2 - 8580\alpha^3 + 17,289.7\alpha^4) \tag{3}$$

In view of Eq. (3), αk_α can be written as $\alpha k_\alpha = \alpha k_{\alpha_0} + k_{n_\alpha}$, where $k_{n_\alpha} = k_{\alpha_1}\alpha^2 + k_{\alpha_2}\alpha^3 + k_{\alpha_3}\alpha^4 + k_{\alpha_4}\alpha^5$ is a nonlinear function of α. The constant parameters $k_{\alpha i}$, $i = 0, 1, \ldots, 4$ can be obtained using Eq. (3).

Defining the state vector $q = (\alpha, h, \dot{\alpha}, \dot{h})^T$, one obtains a state variable representation of Eq. (1) in the form

$$\dot{q} = \begin{bmatrix} 0_{2\times 2} & I_{2\times 2} \\ M_1 & M_2 \end{bmatrix} q + \begin{bmatrix} 0_{2\times 1} \\ g \end{bmatrix} k_{n_\alpha}(\alpha) + \begin{bmatrix} 0_{2\times 1} \\ b_0 \end{bmatrix}\beta \doteq f(q) + b\beta \tag{4}$$

where $b = (b_{01}, b_{02})^T$, $g = (g_1, g_2)^T$, 0 and I denote null and identity matrices of appropriate dimensions, the superscript T denotes transposition, and

$$M_1 = \begin{bmatrix} -\left(k_4U^2 + md^{-1}k_{\alpha_0}\right) & -k_3 \\ -\left(k_2U^2 - mx_\alpha b_s d^{-1}k_{\alpha_0}\right) & -k_1 \end{bmatrix}$$

$$M_2 = \begin{bmatrix} -c_{41} & -c_{31} \\ -c_{21} & -c_{11} \end{bmatrix}$$

The expressions for the parameters k_i,d,c_{ij}, g_i, and b_{0i} are given in Appendices A and B. It is assumed here that the system matrices M_i, g, b, and the structure of the nonlinear function $k_{n\alpha}$ are unknown to the designer. Such an assumption is meaningful because in general it is difficult to model structural nonlinearities. Furthermore, only the signal α or h is available for feedback.

Consider a reference trajectory y_r that represents either a prescribed pitch-angle trajectory α_r for the pitch-angle control or a plunge-displacement trajectory h_r for the plunge motion control. Appropriate reference trajectories converging to zero are generated by a third-order command generator. We are interested in deriving output feedback adaptive control systems so that α tracks α_r or h tracks h_r asymptotically, and in the closed-loop system the state vector $(\alpha, h, \dot{\alpha}, \dot{h})^T$ converges to zero as $t \to \infty$.

III. Adaptive Control Laws

In this section control systems are designed for the trajectory control of the pitch angle and plunge displacement.

A. Pitch-Angle Control

First control of the pitch angle is considered. The design of a control system is based on the modeling error compensation approach of Ref. 20 using a high-gain observer.[21–23] For the pitch-angle control consider an output variable

$$y = \alpha = c(q) = [1 \quad 0 \quad 0 \quad 0]q \tag{5}$$

For the feedback linearization of the input (β)-output (α) map, one differentiates the output y along the trajectory of Eq. (4) until the control input appears for the first time. Differentiating y and using Eq. (4) gives

$$\dot{y} = L_f(c)(q), \qquad \ddot{y} = L_f^2 c(q) + L_b L_f c(q)\beta \tag{6}$$

where $L_f c(q)$ is the Lie derivative of $c(q)$ along the vector field $f(q)$ and

$$L_f c(q) = \left[\frac{\partial c(q)}{\partial q}\right] f(q), \qquad L_b L_f c(q) = \left[\frac{\partial L_f c(q)}{\partial q}\right] b = b_{01}$$

For the system (4)

$$L_f^2 c(q) = \left[M_{1(1)}, M_{2(1)}\right] q + g_1 k_{n_\alpha}(\alpha)$$

where $M_{k(i)}$ is the ith row of M_k.

Using b_{01} given in Appendix B, one finds that it is zero for a unique value x_α^* of x_α, where $x_\alpha^* = -c_{m\beta}/c_{l\beta}$. It is assumed here that $x_\alpha \neq x_\alpha^*$. In view of Eq. (6), one obtains a feedback linearizing control law of the form

$$\beta = [L_b L_f c(q)]^{-1}\left[-L_f^2 c(q) + \ddot{y}_r - p_2\dot{\tilde{y}} - p_1\tilde{y}\right] \tag{7}$$

where $p_i > 0$ and $\tilde{y} = y - y_r$ is the tracking error. Substituting the control law Eq. (7) in Eq. (6) gives

$$\ddot{\tilde{y}} + p_2\dot{\tilde{y}} + p_1\tilde{y} = 0 \tag{8}$$

Because the system Eq. (8) is stable, the tracking error $\tilde{y}$ converges to zero as t tends to ∞. It is seen that Eq. (8) is obtained only if the system parameters are completely known. When there is uncertainty in the system, exact cancellation of functions in Eq. (6) is not possible, and one cannot obtain the linear equation for the tracking error given in Eq. (8).

Now we consider compensation of the unknown functions in Eq. (6) by a proper modification in the control law Eq. (7). We intend to obtain a control law that accomplishes asymptotic feedback linearization. Suppose that

$$L_f^2 c(q) = a^*(y) + \Delta a(q), \qquad L_b L_f c(q) = d^* + \Delta d$$

where a^* and d^* are the nominal values, and Δa and Δd are the unknown parts of $L_f^2 c$ and $L_b L_f c$, respectively. Note that d^* and Δd are constants.

Using Eq. (6) gives

$$\ddot{\tilde{y}} = a^*(y) + \eta(q, \beta) - \ddot{y}_r + d^*\beta \tag{9}$$

where

$$\eta(q, \beta) = \Delta a(q) + \beta\Delta d \tag{10}$$

The function η includes all of the terms of Eq. (6), which are not known.

In view of Eq. (9), for the feedback linearization of the input (β)-output (α) map one must obtain an estimate of the unknown function η. Also, because only α is measured, in view of Eq. (7), one must reconstruct $\dot{\alpha}$ for feedback. One does not have to construct the estimates of h and $\dot{h}$ because an estimate of η is used in the control law.

For obtaining an estimate of η, it is treated as a state variable. In view of Eqs. (9) and (10), consider the set of equations

$$\left(\frac{\mathrm{d}}{\mathrm{d}t}\right)\begin{pmatrix}\tilde{y}\\ \dot{\tilde{y}}\\ \eta\end{pmatrix} = \begin{pmatrix}\dot{\tilde{y}}\\ a^*(y) + \eta(q, \beta) - \ddot{y}_r + d^*\beta\\ f_\eta(q, \beta, \dot{\beta})\end{pmatrix} \tag{11}$$

where the derivative of η is

$$f_\eta = \Delta\dot{a}(q) + \dot{\beta}\Delta d = L_f\Delta a(q) + \beta L_b\Delta a(q) + \dot{\beta}\Delta d \tag{12}$$

For constructing estimates of η and $\dot{\alpha}$, a high-gain observer[21–23] is designed. The advantage of using this observer is that the estimation error converges to zero in a very short period.

In view of Eq. (11), consider an observer of the form

$$\dot{z}_1 = z_2 + \epsilon^{-1}d_1(\tilde{y} - z_1)$$
$$\dot{z}_2 = \hat{\eta} + \epsilon^{-2}d_2(\tilde{y} - z_1) - \ddot{y}_r + a^*(y) + d^*\beta$$
$$\dot{\hat{\eta}} = \epsilon^{-3}d_3(\tilde{y} - z_1) \tag{13}$$

where $d_i > 0$; ϵ is a small positive design parameter; and z_1, z_2, and $\hat{\eta}$ are the estimates of $\tilde{y}$, $\dot{\tilde{y}}$, and the unknown function η, respectively. In the observer Eq. (13) the unknown function f_η is not used. The parameters d_i are chosen such that the polynomial

$$\Pi(s) = s^3 + d_1 s^2 + d_2 s + d_3$$

has its roots with negative real parts.

Defining $e_1 = \tilde{y} - z_1$, $e_2 = \dot{\tilde{y}} - z_2$, and $e_3 = \eta - \hat{\eta} = \tilde{\eta}$, one obtains the dynamics of the estimation error of the form

$$\dot{e}_1 = e_2 - \epsilon^{-1}d_1 e_1, \qquad \dot{e}_2 = e_3 - \epsilon^{-2}d_2 e_1$$
$$\dot{e}_3 = -\epsilon^{-3}d_3 e_1 + f_\eta \tag{14}$$

In view of Eq. (9), using the estimate of the unknown function η one obtains a feedback linearizing control law of the form

$$\beta = (1/d^*)\left[-a^*(y) - \hat{\eta} + \ddot{y}_r - p_2 z_2 - p_1\tilde{y}\right] \tag{15}$$

in which the estimated signal z_2 instead of $\dot{\tilde{y}}$ is used. Substituting Eq. (15) in Eq. (9) gives

$$\ddot{\tilde{y}} = \tilde{\eta} - p_2 z_2 - p_1\tilde{y} \tag{16}$$

Because the input appears in the second derivative of the output, the relative degree of y is two, and there exist zero dynamics of dimension two.

Define new coordinates as

$$x = (x_1, x_2)^T = (\tilde{y}, \dot{\tilde{y}})^T, \qquad v = (h, b_{02}\dot{\alpha} - b_{01}\dot{h})^T \tag{17}$$

where $v = (v_1, v_2) \in R^2$ is the state vector of the zero dynamics. The derivative of v is independent of β. This choice of v yields a normal form [24,25] of Eq. (4). The transformation $\Phi: q \mapsto (x^T, v^T)^T \in R^4$ is a global diffeomorphism, and $q = \Phi^{-1}(x, v)$ in terms of the new variables is given by

$$q = \begin{pmatrix} x_1 + y_r\\ v_1\\ x_2 + \dot{y}_r\\ [b_{02}(x_2 + \dot{y}_r) - v_2]/b_{01}\end{pmatrix} \tag{18}$$

Introduce a change of variables as ($i = 1, 2, 3$)

$$\xi_i = e_i\epsilon^{i-3} \tag{19}$$

One can obtain a new representation of the closed-loop system in the form

$$\dot{x} = \begin{pmatrix} x_2 \\ -p_2 z_2 - p_1 x_1 + \tilde{\eta} \end{pmatrix} = \begin{pmatrix} x_2 \\ -p_2 x_2 - p_1 x_1 + \xi_3 + \epsilon p_2 \xi_2 \end{pmatrix}$$

$$\doteq A_x x + F_x(\epsilon, \xi) \tag{20}$$

$$\dot{v} = \begin{pmatrix} b_{01}^{-1}[b_{02}(x_2 + \dot{y}_r) - v_2] \\ (b_{02}, -b_{01})\left[(M_1, M_2)\Phi^{-1}(x, v) + g k_{n\alpha}\right] \end{pmatrix}$$

$$\doteq A_v v + F_v(x, Y_r) \tag{21}$$

where $z_2 = x_2 - \epsilon \xi_2$ has been used for substitution, Y_r denotes y_r and its derivatives, and $F_x = (0, 1)^T(\epsilon p_2 \xi_2 + \xi_3)$. The matrices A_x and A_v are

$$A_x = \begin{pmatrix} 0 & 1 \\ -p_1 & -p_2 \end{pmatrix}$$

$$A_v(U, a) = \begin{pmatrix} 0 & -(1/b_{01}) \\ k_1 b_{01} - k_3 b_{02} & b_{02}(1/b_{01})c_{31} - c_{11} \end{pmatrix}$$

Here $F_v = (F_{v1}, F_{v2})^T$, $F_{v1} = (b_{02}/b_{01})(x_2 + \dot{y}_r)$, and $F_{v2} = r_0 + (x_1 + y_r)r_1 + (x_2 + \dot{y}_r)r_2$, where

$$r_0 = (b_{02}g_1 - b_{01}g_2)k_{n\alpha}$$

$$r_1 = b_{01}\left(k_2 U^2 - m x_\alpha b_s d^{-1} k_{\alpha 0}\right) - b_{02}\left(k_4 U^2 + m d^{-1} k_{\alpha 0}\right)$$

$$r_2 = b_{01}c_{21} + b_{02}(c_{11} - c_{41}) - b_{02}^2(c_{31}/b_{01})$$

The variable v is described by a linear differential equation which has $(x, y_r, \dot{y}_r)$ as its input. Setting (x, Y_r) to zero in Eq. (21) gives the zero dynamics

$$\dot{v} = A_v v \tag{22}$$

Let the elements of A_v be a_{vij}, $i, j = 1, 2$. Then its eigenvalues are

$$\lambda_{1,2} = 0.5\left[a_{v22} \pm \left(a_{v22}^2 + 4a_{v12}a_{v21}\right)^{\frac{1}{2}}\right]$$

These eigenvalues are functions of the flow velocity U and the parameter a. It has been shown in Refs. 13 and 16 that for a given flow velocity the zero dynamics have stable poles (stable eigenvalues of A_v) for a set of values of a. It will be assumed here that the values of U and a are such that the matrix A_v is Hurwitz. Thus the equilibrium point $v = 0$ of the zero dynamics is globally exponentially stable. Also, the solution of Eq. (21) is bounded for any bounded function $(x, y_r, \dot{y}_r)$.

Using the definition of ξ_i, Eq. (14) gives

$$\epsilon\dot{\xi} = A_0 \xi + (0, 0, 1)^T \epsilon f_\eta \tag{23}$$

where $\xi = (\xi_1, \xi_2, \xi_3)^T \in R^3$ and

$$A_0 = \begin{pmatrix} -d_1 & 1 & 0 \\ -d_2 & 0 & 1 \\ -d_3 & 0 & 0 \end{pmatrix}$$

It is essential to expand the function f_η, which is a function of β and its derivative. For this purpose one obtains the expressions for β and $\dot{\beta}$ as functions of (x, v, ξ, Y_r).

Using $z_2 = x_2 - \epsilon\xi_2$ and

$$\hat{\eta} = \eta - \tilde{\eta} = \Delta a(q) + \beta \Delta d - \xi_3$$

in Eq. (15) and solving for β gives

$$\beta[x, v, Y_r, N_0(\epsilon)\xi] = (d^* + \Delta d)^{-1}\left[-a^* - \Delta a + \ddot{y}_r - p_2 x_2 - p_1 x_1 + N_0(\epsilon)\xi\right] \tag{24}$$

provided that

$$|(\Delta d/d^*)| < 1$$

where $N_0(\epsilon) = (0, \epsilon p_2, 1)$. Here $\Phi^{-1}(x, v)$ has been substituted for q in Eq. (24). It is assumed that the uncertainty in Δd satisfies this inequality.

Differentiating β and using $\dot{\xi}_3$ from Eq. (23) and f_η from Eq. (12) gives

$$\dot{\beta} = (d^* + \Delta d)^{-1}\left[-\dot{a}^* - \Delta\dot{a} - \epsilon^{-1}d_3\xi_1 + \Delta\dot{a} + \dot{\beta}\Delta d + \left(d^3 y_r \big/ dt^3\right) - p_2(\dot{x}_2 - \epsilon\dot{\xi}_2) - p_1 x_2\right] \tag{25}$$

Premultiplying Eq. (25) by $(d^* + \Delta d)$, it is seen that one can cancel the Δd-dependent term. Define

$$\mu_1(x, v, Y_r, N_1(\epsilon)\xi) = (d^*)^{-1}\left[-\dot{a}^* + \left(d^3 y_r \big/ dt^3\right) + p_2(p_2 x_2 + p_1 x_1) - p_1 x_2 - N_1(\epsilon)\xi\right] \tag{26}$$

where $N_1(\epsilon) = (p_2 d_2, \epsilon p_2^2, 0)$ and $\dot{a}^* = (\partial a^*/\partial\alpha)(x_2 + \dot{y}_r)$. Substituting for $\dot{x}$ and $\dot{\xi}$ from Eqs. (20) and (23) in (25), replacing α by $x_1 + y_r$, and then solving for $\dot{\beta}$ gives

$$\dot{\beta}(x, v, Y_r, N_1(\epsilon)\xi) = -(d^*)^{-1}\epsilon^{-1}d_3\xi_1 + \mu_1(x, v, Y_r, N_1(\epsilon)\xi) \tag{27}$$

Equations (24) and (27) give explicit solutions for β and $\dot{\beta}$.

Using f_η from Eq. (12) and $\dot{\beta}$ from Eq. (27) in Eq. (23) gives

$$\epsilon\dot{\xi}_3 = -d_3\xi_1 + \epsilon\left\{L_f \Delta a + \beta L_b \Delta a + \Delta d\left[-d_3(d^*\epsilon)^{-1}\xi_1 + \mu_1\right]\right\} \tag{28}$$

Using Eq. (28) in Eq. (23), replacing q by $\Phi^{-1}(x, v)$, and substituting for β from Eq. (24) gives

$$\epsilon\dot{\xi} = A_f \xi + (0, 0, 1)\epsilon\mu_f(x, v, Y_r, N_2(\epsilon)\xi) \tag{29}$$

where $N_2(\epsilon)$ is an appropriate polynomial row vector in ϵ, $\mu_f = \Delta d \mu_1 + L_f \Delta a + \beta L_b \Delta a$, and A_f is

$$A_f = \begin{pmatrix} -d_1 & 1 & 0 \\ -d_2 & 0 & 1 \\ -d_3[1 + (\Delta d/d^*)] & 0 & 0 \end{pmatrix}$$

It is noted that μ_f is a function of the indicated arguments.

The eigenvalues of A_f are the roots of equation

$$\Pi_f(s) = s^3 + d_1 s^2 + d_2 s + \left(d_3 \big/ d^*\right)(d^* + \Delta d) = 0 \tag{30}$$

Applying the Routh–Hurwitz criterion, it follows that A_f is Hurwitz if and only if

$$(d_1 d_2/d_3) > 1 + (\Delta d/d^*), \qquad |(\Delta d/d^*)| < 1 \tag{31}$$

It is assumed that the design parameters d_i have been chosen to satisfy these conditions.

For small ϵ the closed-loop system represented by Eqs. (20), (21), and (29) is a singularly perturbed system with $x_s = (x^T, v^T)^T$ as the slow variable and ξ as the fast one. For $\epsilon = 0$ Eq. (29) degenerates into the equation

$$A_f \xi = 0 \tag{32}$$

which has a unique solution $\xi = 0$.

Noting that one is interested in the stabilization of the aeroelastic system, one can choose $y_r = 0$. Therefore, it will be assumed for simplicity in the remaining part of this section that the reference trajectory y_r is zero, and therefore one can set $Y_r = 0$ as well. Substituting the quasi-steady state $\xi = 0$ and using $Y_r = 0$ in Eqs. (20) and (21) gives the reduced model

$$\dot{x} = \begin{pmatrix} x_2 \\ -p_2 x_2 - p_1 x_1 \end{pmatrix} = A_x x, \qquad \dot{v} = A_v v + F_v(x, 0) \tag{33}$$

Because A_x and A_v are Hurwitz, in view of Eq. (33) it follows that the equilibrium state $x_s = 0$ of the system (33) is exponentially stable. Choosing a new time variable $\tau = t/\epsilon$, the boundary-layer model obtained from Eq. (29) is given by

$$\frac{\mathrm{d}\xi}{\mathrm{d}\tau} = A_f \xi(\tau) \tag{34}$$

Because A_f is Hurwitz, $\xi = 0$ of Eq. (34) is globally exponentially stable.

Consider a compact domain of interest $\Omega = \Omega_s \times \Omega_\xi \subset R^4 \times R^3$ containing the origin $x_s = 0$ and $\xi = 0$. F_x and μ_f are smooth functions of x_s and ξ, and vanish at the origin if y_r is zero. Because the partial derivatives of F_x and μ_f are bounded on any compact set, in view of the mean value theorem,[26] it follows that for $(x_s, \xi) \in \Omega_s \times \Omega_\xi$ and for all $0 < \epsilon < \epsilon^*$ there exist positive real numbers L_i such that

$$\|F_x(\epsilon, \xi)\| \le L_1 \|\xi\|$$

$$\|\mu_f[x, v, 0, N_2(\epsilon)\xi]\| \le L_2\|x_s\| + L_3\|\xi\| \tag{35}$$

where ϵ^* is a small positive number. (Exact values of L_i are not required for establishing stability results.)

In view of the exponential stability of the reduced system Eq. (33) and the boundary-layer system Eq. (34) and the growth conditions given in Eq. (35), the following result for the singularly perturbed system represented by Eqs. (20), (21), and (29) is stated.

Theorem 1: Consider the singularly perturbed system Eqs. (20), (21), and (29). Suppose that for the given flow velocity U and the parameter a, A_v is Hurwitz, and the uncertainty Δd in $L_b L_f c(q)$ and the growth conditions satisfy Eqs. (31) and (35), respectively. Then there exist a region $\Omega_0 \subset \Omega$ and a critical value ϵ^* such that for all initial conditions $[x_s(0), \xi(0)] \in \Omega_0$ and $0 < \epsilon < \epsilon^*$ the state $[x_s(t), \xi(t)]$ asymptotically converges to zero. (Under the hypotheses of the theorem, the proof is completed invoking a Tikhonov-like theorem derived in Ref. 21.)

For the aeroelastic model Eq. (4) the singularly perturbed Eqs. (20), (21), and (29) have been obtained by using the control law Eq. (15) and the observer Eq. (13) and by introducing change of coordinates given in Eqs. (18) and (19). The growth condition of Theorem 1 required in the proof is automatically satisfied by the smoothness of the functions in the model. Thus one needs only to satisfy Eq. (31) and verify that the zero dynamics are stable (i. e., A_v is Hurwitz). It is pointed out that Eq. (31) can be satisfied if Δd (the uncertainty in b_{01}) is small and that stability of the zero dynamics is essential even in the nonadaptive design.[13] According to Theorem 1, as (x_s, ξ) converges to zero, in view of the transformation $q = \Phi(x, v) = \Phi(x_s)$ [Eq. (18)], the pitch angle and plunge displacement also asymptotically tend to zero.

The complete closed-loop system is shown in Fig. 2. The control system consists of the control law Eq. (15) and the high-gain observer Eq. (13). Equation (15) is essentially a feedback linearizing control law obtained by the inversion of the $\beta - \alpha$ map, in which estimated values $\hat{\eta}$ and z_2 of the modeling error term η and $(\dot{\tilde{y}} = \dot{\alpha} - \dot{y}_r)$, respectively, are used for feedback. The observer reconstructs the modeling error term η and $\dot{\tilde{y}} = \dot{\alpha} - \dot{y}_r$ for the synthesis of the control law. Because the observer has a high-gain feedback loop, stability proof is based on the singular perturbation theory.

The ϵ-dependent scaling causes an impulsive-like behavior in ξ as ϵ tends to zero. This is avoided by saturating the control input if it exceeds its bounds. Using saturating control input also, one can follow the steps of Refs. 21–23 to establish stability in the closed-loop system. It is seen in the next section that for the chosen values of ϵ control remains well within its bounds, and, therefore, one does not need a saturating control law for stabilization. At this point, it is useful to discuss how the control system works. For small ϵ the fast variable ξ decays close to zero in a short transient period, but the slow variable (x, v) remains near its initial value. After this short boundary-layer period the estimate $(z, \hat{\eta})$ becomes very close to the state (x, η), and the inverse control system recovers the performance of the state feedback linearizing controller of the model with known dynamics. This feature of controller is also preserved even if the control input saturates because the period of saturation is extremely small when the fast variable decays rapidly.

B. Plunge Motion Control

For plunge motion control one chooses the output $y = h$ and $y_r = h_r$. In this case, following the approach of the preceding section one obtains the control law based on the $\ddot{h}$ equation. It is easily seen that in this case η is caused by the uncertainty in $M_{i(2)}$, $g_2 k_{n\alpha}$, and b_{02}. In the observer and the control law one uses $h - h_r$ for feedback. Because the design can be easily completed following Sec. III.A, the details are not presented here.

Remark 1: The observer uses high-gain feedback to compensate for the uncertain function f_η in the estimation error dynamics Eq. (14). Therefore, the study of the effect of measurement noise on the closed-loop system stability is important. But this problem has not yet received much attention. Interestingly, it will be seen in the next section by simulation that the closed-loop system is sufficiently robust, and stability is preserved even in the presence of measurement noise.

Remark 2: For the aeroelastic model it is seen that the pitch angle and the plunge displacement are of relative degree two. However, it is pointed out that this design approach can be easily extended to minimum phase systems of arbitrary dimension for which the output variable is of higher relative degree. For an output with relative degree r, one constructs an observer similar to Eq. (13) but of dimension $(r + 1)$. In this case the uncertain function η is given by $\eta = \Delta L_f^r c(q) + u_i \Delta L_b L_f^{r-1} c(q)$, where u_i is the input and the terms with Δ are unknown functions in the rth derivative of the output $y = c(q)$.

IV. Simulation Results

The open-loop system Eq. (1) for $U = 15$ m/s and $a = -0.4$ with initial condition $\alpha(0) = 5.7296$ (deg), $\dot{\alpha}(0) = 0$, $h(0) = 0.01$ (m), and $\dot{h}(0) = 0$ is simulated. Figure 3 shows the oscillatory response of the system.

Now the results of digital simulation for the aeroelastic model Eq. (1) with the adaptive control law Eq. (15) including the observer Eq. (13) are presented. The initial conditions of Fig. 3 are assumed. A third-order reference model

$$\left(s^3 + 3\lambda_c s^2 + 3\lambda_c^2 s + \lambda_c^3\right) y_r = 0 \tag{36}$$

generates the reference trajectory y_r converging to zero for trajectory tracking, where $\lambda_c = 1.5$. For the pitch-angle control $y_r = \alpha_r$, and the initial conditions are $y_r(0) = \alpha(0)$ and $\ddot{y}_r(0) = \dot{y}_r(0) = 0$, but for the plunge-displacement control $y_r = h_r$, $y_r(0) = h(0)$, and $\ddot{y}_r(0) = \dot{y}_r(0) = 0$.

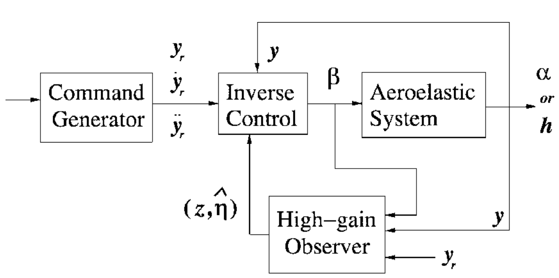


Fig. 2 Closed-loop system.

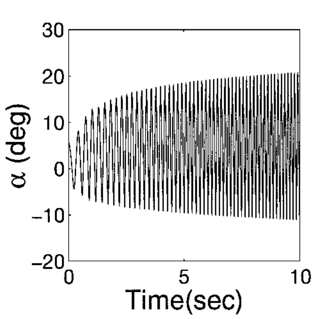
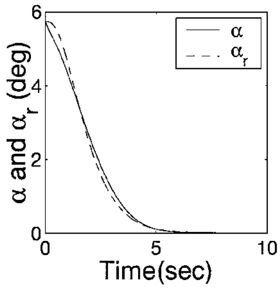
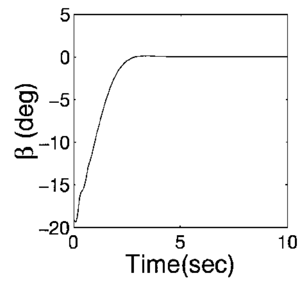


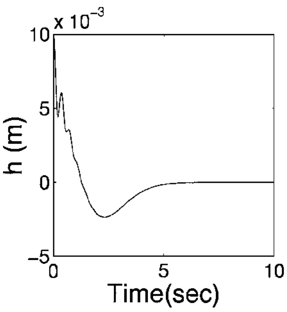
Fig. 3 Open-loop response.



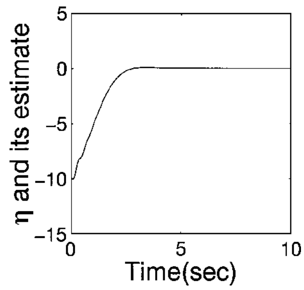
a) **Pitch angle α and α_r**



c) **Control input β**



b) **Plunge displacement h**



d) **Uncertain function η and its estimate $\hat{\eta}$**

Fig. 4 Pitch-angle control.

For the pitch-angle control only α is measured, and for the plunge-motion control only h is available for synthesis. The feedback gains in control law Eq. (15) are $p_2 = 2\zeta w_n$, $p_1 = w_n^2$, $\zeta = 0.707$, and $w_n = 1.5$. The gains for the observer are selected to be $d_1 = 3$, $d_2 = 3$, and $d_3 = 1$, and its initial conditions are chosen as $z_1(0) = z_2(0) = 0$, $\hat{\eta}(0) = 0$. The design parameters have been selected by observing the simulated responses.

For simulation it is assumed that the function $L_f^2 c(q)$ is completely unknown, and therefore its nominal value a^* is taken to be zero. Such a worse choice of a^* is made only to show robustness of the designed controller. The nominal value d^* is assumed to be $0.8b_{01}$, which gives an uncertainty of 20% in b_{01}. For this uncertainty in b_{01} and chosen observer parameters d_i, Eq. (31) is satisfied, and A_f is Hurwitz.

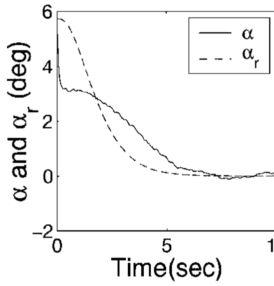
A. Pitch-Angle Control

A reference trajectory $y_r = \alpha_r$ is generated for the pitch-angle control. Figures 4a–4d show the selected responses of the closed-loop system for $U = 15$ m/s, $a = -0.4$, and $\epsilon = 0.001$. The zero dynamics are stable for this choice of U and a. The pitch angle asymptotically follows the reference trajectory $y_r = \alpha_r$. The response time for α of the order of 5 s is observed. The flap deflection is less than 20 deg. Because the zero dynamics are stable, the plunge motion also converges to zero. Interestingly, the high-gain observer causes fast convergence of $\hat{\eta}(t)$ to the uncertain function $\eta(t)$ without any overshoot. Even though $\eta(0) \neq \hat{\eta}(0)$, η and $\hat{\eta}$ responses overlap after an extremely short boundary-layer interval.

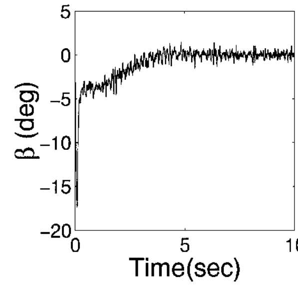
The parameter ϵ of the observer has a significant effect on the response characteristics of the system. It is found by simulation that the tracking and regulation performance improves as the value of ϵ decreases. The reason for this is that the estimator becomes quicker in estimating the uncertain function as ϵ decreases. But smaller ϵ can lead to a large swing in the estimated states and amplification of the measurement noise in the feedback loop, which can cause frequent control saturation. Also it is noted that if ϵ exceeds a critical value the system becomes unstable.

B. Pitch-Angle Control: Nonzero Measurement Noise

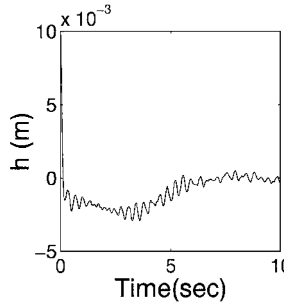
To examine the effect of measurement noise on the performance of the control system, simulation is done by introducing a measurement noise in the pitch angle. Thus in the estimator Eq. (13), $\tilde{y}$ is replaced by $[\alpha + n_\alpha(t) - \alpha_r]$, where n_α is the measurement noise. Also in the control law Eq. (15), $\tilde{y}$ is replaced by its estimate z_1. The command trajectory y_r, initial conditions, and feedback gains of case A are retained, but ϵ is set to 0.017. Simulated responses are shown in Figs. 5a–5e. The maximum value of the noise exceeds 0.15 deg. It is seen that in spite of the measurement noise and model uncertainty the adaptive controller including the observer accomplishes trajectory control of α and stabilization of the state vector close to the origin. The response time of the pitch angle is of the order of 8 s, which is larger than that of case A because of the presence of noise. The high-gain feedback makes the observer sufficiently robust, but the control input experiences oscillations of small magnitude because it tries to cancel the effect of noise on the pitch angle. Of course, if the measurement noise is smoother, these oscillations in the control input will reduce. It is found that for smaller ϵ the tracking performance improves, but the control input undergoes faster oscillations. (These results are not shown here in order to save space.)



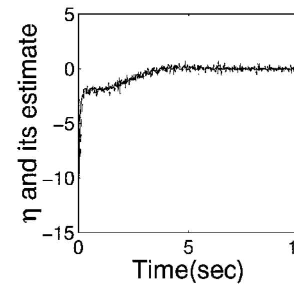
a) **Pitch angle α and α_r**



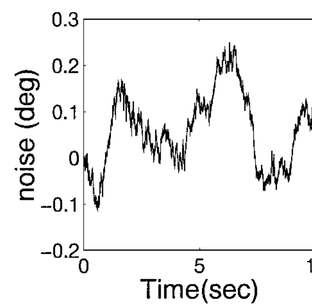
c) **Control input β**



b) **Plunge displacement h**



d) **Uncertain function η and its estimate $\hat{\eta}$**



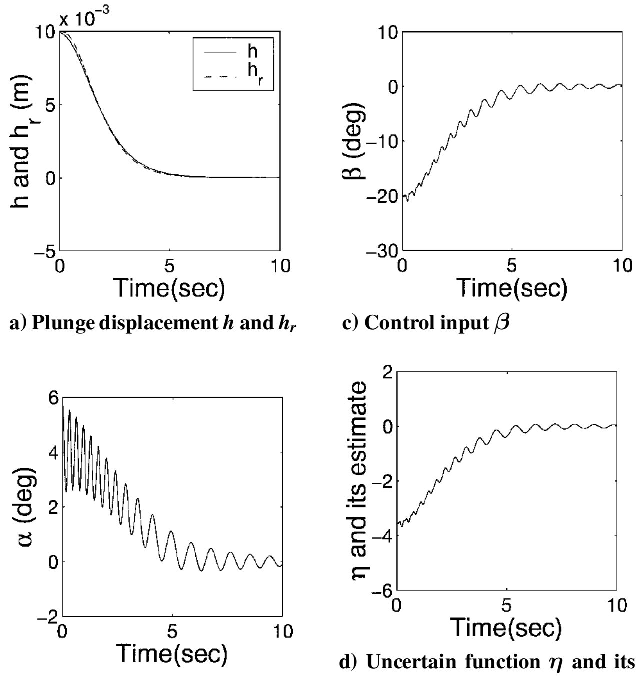
e) **Measurement noise**

Fig. 5 Pitch-angle control: nonzero measurement noise.

C. Plunge-Motion Control

For plunge-motion control the closed-loop system for $U = 15$ m/s and $a = -0.68$ is simulated. For these values of U and a, the zero dynamics are stable. The feedback gains and initial conditions of case A are retained. In this case control system uses only signal h for feedback. Responses for the observer parameter $\epsilon = 0.0001$ are shown in Figs. 6a–6d. We observe good trajectory following of $h(t)$, but α, β, η, and $\hat{\eta}$ have oscillatory responses.

Unlike α control, here one controls the plunge-displacement trajectory. When the plunge displacement vanishes, the residual (zero) dynamics are governed by complicated nonlinear differential equations because the wing model includes α-dependent nonlinear structural nonlinearity. These oscillations in α are caused by the nonlinear zero dynamics. However, it is seen that these oscillations gradually decay to zero and the state vector attains the equilibrium null state.



a) Plunge displacement h and h_r

c) Control input β

b) Pitch angle α

d) Uncertain function η and its estimate $\hat{\eta}$

Fig. 6 Plunge-motion control.

In this case the maximum value of β is about 21 deg. As in Fig. 4, $\hat{\eta}$ converges almost instantaneously to η.

Extensive simulations have been performed for pitch-angle and plunge-displacement control. These results show that in the closed-loop system robust trajectory tracking and stabilization are accomplished by the designed controllers.

V. Conclusions

In this paper a new control system for the control of an aeroelastic system based on a modeling error compensation technique was presented. Adaptive control laws for the trajectory control of the pitch angle and plunge displacement were derived. For the synthesis of the controller, only the measured variable (plunge displacement or pitch angle) was used. The structure of the control system is similar to a feedback linearizing controller. The uncertain function in the control law was compensated using its estimate obtained by a high-gain observer. In the closed-loop system asymptotic regulation of the state vector to the origin was accomplished. Simulation results were presented, which show that control of the pitch angle and the plunge displacement can be accomplished using only output feedback in spite of the uncertainties in the system. Moreover, the observer is sufficiently robust, and the closed-loop system remains stable in the presence of measurement noise.

Simulation results show that the design parameter ϵ used in the observer plays an important part in shaping the closed-loop responses. As the parameter ϵ decreases, the trajectory tracking and stabilization improve, but this can cause peaking (overshoot) phenomenon in the estimator states and amplify the measurement noise transmitted into the feedback loop. This in turn can cause control saturation. Of course, stability in the closed-loop system is lost if ϵ exceeds a critical value. For the aeroelastic system the chosen values of ϵ lead to accurate trajectory tracking without any peaking phenomenon and control saturation.

Appendix A: System Parameters

$$b_s = 0.135 \text{ m}, \qquad k_h = 2844.4 \text{ N/m}, \qquad c_h = 27.43 \text{ Ns/m}$$

$$c_\alpha = 0.036 \text{ Ns}, \qquad \rho = 1.225 \text{ kg/m}^3, \qquad c_{l\alpha} = 6.28$$

$$c_{l\beta} = 3.358, \qquad c_{m\alpha} = (0.5 + a)c_{l\alpha}, \qquad c_{m\beta} = -0.635$$

$$m = 12.387 \text{ kg}, \qquad I_\alpha = 0.065 \text{ kgm}^2$$

$$x_\alpha = [0.0873 - (b_s + ab_s)]/b_s$$

Appendix B: System Variables

$$d = m\left(I_\alpha - mx_\alpha^2 b_s^2\right), \qquad k_1 = I_\alpha k_h/d$$

$$k_2 = \left(I_\alpha \rho b_s c_{l\alpha} + mx_\alpha b_s^3 \rho c_{m\alpha}\right)\big/d, \qquad k_3 = -mx_\alpha b_s k_h/d$$

$$k_4 = \left(-mx_\alpha b_s^2 \rho c_{l\alpha} - m\rho b_s^2 c_{m\alpha}\right)\big/d$$

$$c_{11} = \left[I_\alpha(c_h + \rho U b_s c_{l\alpha}) + mx_\alpha \rho U b_s^3 c_{m\alpha}\right]\big/d$$

$$c_{21} = \left[I_\alpha \rho U b_s^2 c_{l\alpha}\left(\tfrac{1}{2} - a\right) - mx_\alpha b_s c_\alpha + mx_\alpha \rho U b_s^4 c_{m\alpha}\left(\tfrac{1}{2} - a\right)\right]\big/d$$

$$c_{31} = \left(-mx_\alpha b_s c_h - mx_\alpha \rho U b_s^2 c_{l\alpha} - m\rho U b_s^2 c_{m\alpha}\right)\big/d$$

$$c_{41} = \left[mc_\alpha - mx_\alpha \rho U b_s^3 c_{l\alpha}\left(\tfrac{1}{2} - a\right) - m\rho U b_s^3 c_{m\alpha}\left(\tfrac{1}{2} - a\right)\right]\big/d$$

$$b_{01} = U^2\left(mx_\alpha b_s^2 \rho c_{l\beta} + m\rho b_s^2 c_{m\beta}\right)\big/d$$

$$b_{02} = U^2\left(-I_\alpha \rho b_s c_{l\beta} - mx_\alpha b_s^3 \rho c_{m\beta}\right)\big/d, \qquad g = \begin{bmatrix} -m/d \\ mx_\alpha b_s/d \end{bmatrix}$$

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